SHORT COMMUNICATION

# EXTENSION OF FINITE LENGTH SIGNALS FOR SUB-BAND CODING

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**Abstract.** Some problems associated with sub-band coding of images are investigated. The filtering operations needed for splitting an input into different frequency bands, as well as their reconstruction, require an extension beyond the length of the finite signal. If not properly made, the necessary signal extension can yield distortion of the reconstructed signal, and it may complicate the compression of high-pass frequency bands. Five different methods of signal extensions have been investigated for sub-band coding. It turns out that extending the signal so that continuity is maintained may be a sufficient remedy, as shown in an image coding example.

**Zusammenfassung.** In diesem Artikel beschäftigen wir uns mit einigen Teilproblemen bei der Subbandcodierung von Bildern. Die Filteroperationen, die für die Aufspaltung des Bildes in mehrere Frequenzbänder wie auch für die zugehörige Rekonstruktion gebraucht werden, benötigen Signalwerte jenseits der Grenzen des finiten Bildsignales. Die notwendige Ergänzung des Signals kann Nebeneffekte zur Folge haben, wie z.B. die Verzerrung des rekonstruierten Signals, und sie kann die Komprimierung der höheren Frequenzbänder erschweren. Wir haben insgesamt fünf verschiedene Methoden der Erweiterung des Signals über die Bildgrenzen hinaus unter dem Gesichtspunkt der Subbandcodierung untersucht und die zwei genannten Nebenwirkungen in Betracht gezogen. Wie sich zeigt, genügt es, bei der Erweiterung des Bildsignals die Kontinuität sicherzustellen; dies wird an einem praktischen Beispiel zur Bildcodierung demonstriert.

**Résumé.** Quelques problèmes liés au codage en sous-bandes d'image sont considérés. L'opération de filtrage requise afin de diviser le signal d'entrée en plusieurs bandes de fréquence, ainsi que la reconstruction, fait appel à une extension du signal de longueur finie. Si cette extension n'est pas faite correctement, des distortions peuvent apparaître et la compression possible peut être détériorée dans les bandes de haute fréquence. Cinq méthodes d'extension différentes ont été testées pour le codage en sous-bandes. Il s'avère qu'une extension préservant la continuité du signal est suffisante comme le démontre un example de codage d'image en sous-bandes.

**Keywords.** Sub-band coding, image compression, signal extrapolation.

## 1. Introduction

Sub-band coding has recently received increased attention as a technique for image compression (see for example [1] and [2]). The technique, which can yield high compression and quality, is a strong contender to discrete cosine transform coding by avoiding most of that method's disadvantages, such as blocking effects and unexploited redundancy between blocks. The technique of sub-band coding is explained in Fig. 1 for a two band system. A signal is passed through the analysis filters. Due to the reduced bandwidth of the resulting components they may be subsampled to yield the sub-

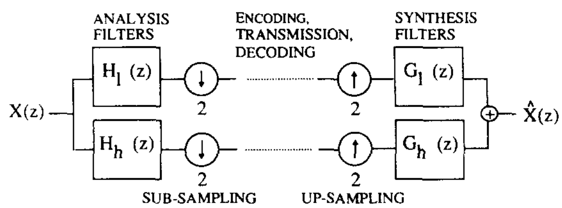


Fig. 1. Sub-band coding system with two bands.

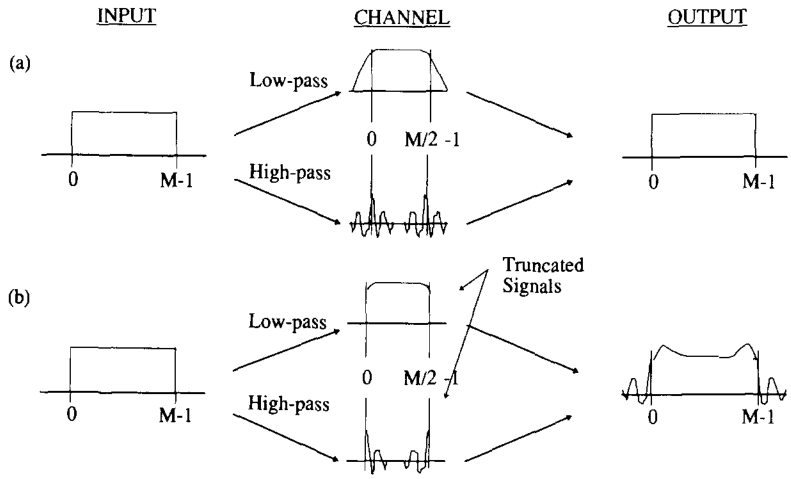


Fig. 2. Finite length sub-band coding: (a) signal with extended length in the channel; (b) signal with maintained length (through truncation) in the channel.

bands. Following that, each sub-band is encoded, transmitted and, at the destination, decoded. To reconstruct the signal, the sub-bands are up-sampled to the sampling rate of the input and passed through the synthesis filter bank. The filtered components are then added to form the reconstructed signal.

Sub-band analysis performed on signals of finite extent, such as images, will result in a total amount of data in the channel greater than that of the input (see Fig. 2(a)). This is generally undesirable in a compression application because of the increase in the number of samples to be coded and transmitted. Also, from an implementation point of view, the sizes of sub-divided images become non-standard. If the number of samples to be encoded is enforced to be equal to that of the input, the analyzed signals must be truncated whereby the information loss may lead to distortion of the reconstructed signal, as illustrated in Fig. 2(b). To alleviate this, an input signal has to be extended in an appropriate way before the analysis filtering so that the information loss is minimized. Following that, the sub-bands are truncated, and, at the receiver, they are extended for the synthesis filtering. Depending on the method of extension, the signal may be distorted by the analysis and the synthesis, and the compression of the signal may be more complicated.

Note that sub-band coding of images, as described in the literature, has used simple extension scheme, like circular extension [1]. An improved solution to the problem has been proposed in [3]. In this paper, we present an investigation of the effects of five different methods for signal extension. The study includes a simulation of the seven band sub-band coding system. Throughout the paper quadrature mirror filters (FIR) are considered, which gives alias free reconstruction.

## 2. The signal extension methods and their side effects

Initially, a one dimensional setting is considered, but the results hold for higher dimensions [4]. Assume that the length of the filter's impulse-response is $N$, and that the signal, $x(n)$, is of length $M$ samples. The input signal is extended by adding $\frac{1}{2}N-1$ values at each end. The following five non-predictive ways of extending the finite signal have been investigated (depicted in Fig. 3):

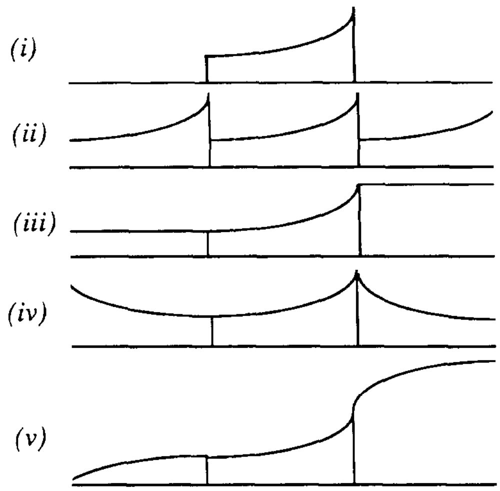


Fig. 3. Methods of signal extension: (i) pad with zeroes; (ii) circular extension; (iii) replication of edge values; (iv) symmetric extension [3]; (v) doubly symmetric extension.

(i) Pad with zeroes; where the signal is assumed to be zero outside its support, so that

$$x(n)=0 \quad \text{for } -\tfrac{1}{2}N+1 \le n<0 \quad \text{and } M \le n \le M+\tfrac{1}{2}N-2.$$

(ii) Circular extension; where the signal is assumed to be periodic with period $M$. Thus,

$$x(n)=x((n+M) \bmod M) \quad \text{for } -\tfrac{1}{2}N+1 \le n<0$$

and

$$x(n)=x(n \bmod M) \quad \text{for } M \le n \le M+\tfrac{1}{2}N-2.$$

(iii) Replication of boundary values, which means that the signal is made continuous at the ends by repeating the first and the last values of the signal respectively. I.e.,

$$x(n)=x(0) \qquad \text{for } -\tfrac{1}{2}N+1 \le n<0$$

and

$$x(n)=x(M-1) \quad \text{for } M \le n \le M+\tfrac{1}{2}N-2.$$

(iv) Symmetric extension [3], which is similar to (ii) by making the signal periodic, but this time by period $2M$. This is achieved by extending the signal by its mirror image, whereby it becomes symmetric around the boundaries. Thus

$$x(n)=x(-n-1) \quad \text{for } -\tfrac{1}{2}N+1 \le n<0$$

and

$$x(n)=x(2M-1-n) \quad \text{for } M \le n \le M+\tfrac{1}{2}N-2.$$

(v) Doubly symmetric extension, which is achieved by flipping the signal not only in time (as (iv)), but also in amplitude, as shown in Fig. 3. This is given by

$$x(n)=2x(0)-x(-n) \quad \text{for } -\tfrac{1}{2}N+1 \le n<0,$$

and

$$x(n)=2x(M-1)-x(2M-1-n) \quad \text{for } M \le n \le M+\tfrac{1}{2}N-2.$$

Note that method (i) and (ii) will in general create a discontinuity in the extended signal, whereas (iii), (iv) and (v) will maintain continuity. For method (v), the first derivative is continuous at the boundary as well. Note that the methods (iv) and (v) need a more complex implementation than the other three. The complexity can, roughly speaking, be considered zero for methods (i)-(iii) (i.e., replication of zeroes, circular addressing, and replication of first and last values, respectively). For method (iv) an address calculation is necessary to replicate the correct values. Finally, method (v) requires one addition and one multiplication, or shift operation, as well as the address calculation.

The input/output relationship will depend on the extension method used. The matrix describing this relationship has been computed for a system with two sub-bands using Johnston filters of length 16 [5]. The results for the methods (i)-(v) are shown in Table 1. The transfer functions show losses for all methods but circular extension. Note that only method (ii) gives aliasing cancellation at the end of a signal. The visibility of this distortion in an image coding application is investigated in the next section.

Table 1

Transmission matrices (input to output), evaluated for the extension methods (i)-(v) using filters of length 16 from [5]. (The lower-right corners of the matrices are analogous.)

(i)
$$\begin{bmatrix} 0.94 & 0 & 0.01 & 0 & 0 & \cdots \\ 0 & 0.95 & 0 & 0.02 & 0 & \cdots \\ 0.01 & 0 & 1.00 & 0 & 0 & \cdots \\ 0 & 0.02 & 0 & 0.99 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 1.00 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \end{bmatrix}$$

(ii)
$$\begin{bmatrix} 1.00 & 0 & 0 & \cdots \\ 0 & 1.00 & 0 & \cdots \\ 0 & 0 & 1.00 & \ddots \\ \vdots & \vdots & \ddots & \ddots \end{bmatrix}$$

(iii)
$$\begin{bmatrix} 0.95 & 0 & 0.06 & 0 & -0.01 & 0 & 0 & \cdots \\ 0.15 & 0.81 & 0 & 0.05 & 0 & -0.02 & 0 & \cdots \\ 0.01 & 0 & 0.99 & 0 & 0 & 0 & 0 & \cdots \\ -0.06 & 0.07 & 0 & 0.98 & 0 & 0.01 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 1.00 & 0 & 0 & \cdots \\ 0.04 & -0.5 & 0 & 0.01 & 0 & 0.99 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \end{bmatrix}$$

(iv)
$$\begin{bmatrix} 1.17 & -0.26 & 0.02 & 0.12 & -0.02 & -0.05 & 0.01 & 0 & \cdots \\ 0.16 & 0.79 & -0.04 & 0.14 & 0.01 & -0.08 & 0.01 & 0 & \cdots \\ -0.04 & 0.06 & 0.99 & -0.02 & 0.02 & -0.01 & 0 & 0 & \cdots \\ -0.07 & 0.10 & 0.01 & 0.93 & 0.01 & 0.02 & 0 & 0 & \cdots \\ 0 & -0.01 & 0.01 & -0.01 & 1.00 & 0 & 0 & 0 & \cdots \\ 0.03 & -0.05 & 0.01 & 0.01 & 0 & 0.99 & 0 & 0 & \cdots \\ 0.01 & -0.02 & 0 & 0 & 0 & 0 & 1.00 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \end{bmatrix}$$

(v)
$$\begin{bmatrix} 0.92 & 0 & 0.19 & 0 & -0.17 & 0 & 0.08 & -0.02 & 0 & \cdots \\ 0.23 & 0.59 & 0 & 0.28 & 0 & -0.16 & 0 & 0.09 & 0 & \cdots \\ 0.01 & 0 & 0.96 & 0 & 0.04 & 0 & -0.01 & 0 & 0 & \cdots \\ -0.09 & 0.16 & 0 & 0.88 & 0 & 0.06 & 0 & -0.02 & 0 & \cdots \\ -0.02 & 0 & 0.02 & 0 & 1.00 & 0 & 0 & 0 & 0 & \cdots \\ 0.05 & -0.09 & 0 & 0.05 & 0 & 0.98 & 0 & 0.01 & 0 & \cdots \\ -0.01 & 0 & 0 & 0 & -0.01 & 0 & 1.00 & 0 & 0 & \cdots \\ -0.01 & 0 & 0.02 & 0 & 0 & -0.01 & 0 & 1.00 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \end{bmatrix}$$

If the extension of the input signal results in a discontinuity it will yield artificially high amounts of energy in the high-pass band. Substantial coding gain in sub-band coding is achieved by high compression of the high-pass band, often through coarse quantization. Thus, the method of signal extension may lead to less compression to avoid quality degradation, or, with fixed compression, distortion of the signal at start and end regions of the signal. To illustrate this effect, an exponential test input has been analyzed into two sub-bands, but reconstructed using only the low-pass band.

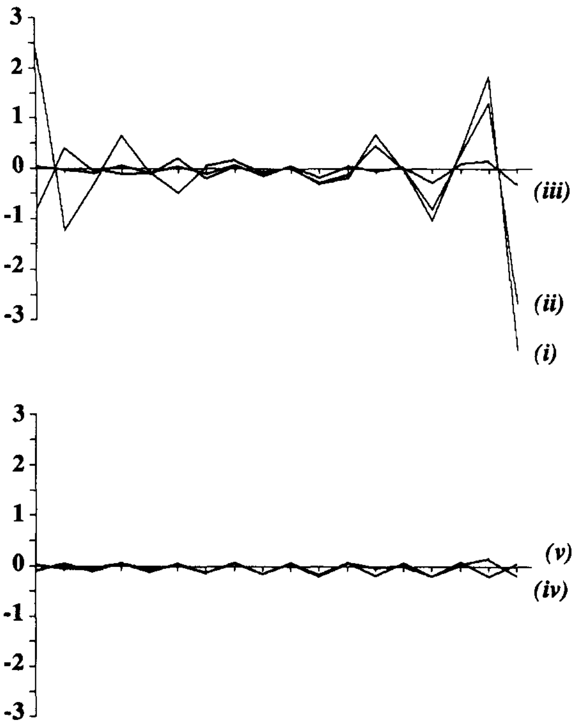


Fig. 4. Reconstruction error when an exponential input signal is reconstructed from the low-pass component. The amplitude of the input is 3.7 at the left end and 15.2 at the right one. (i)-(v) refer to the method of signal extension. The filter used is Johnston's of length 16 [5].

The reconstruction error is shown in Fig. 4. The severity of this issue in a true sub-band coding simulation is discussed in the next section.

## 3. Boundary effects in a sub-band coding system

Test images have been extended following each method, and they were subsequently sub-band encoded and decoded. The analysis was performed with separable filters first horizontally followed by vertical analysis to yield initially four bands. The band which was obtained by low-pass filtering in both dimensions was further analyzed into four bands. Hence, the analysis yields seven bands to be encoded as shown in Fig. 5. The synthesis part was analogous. Johnston filters of length 16 [5] were used for both analysis and synthesis. A similar system is described in more detail in [2].

The effect of the input/output relation was evaluated by performing analysis and synthesis without any coding. The image was extended and analyzed. The seven bands were extended by the same method for the synthesis. There is virtually perfect reconstruction when using pad-with-zeroes, and circular extension (methods (i) and

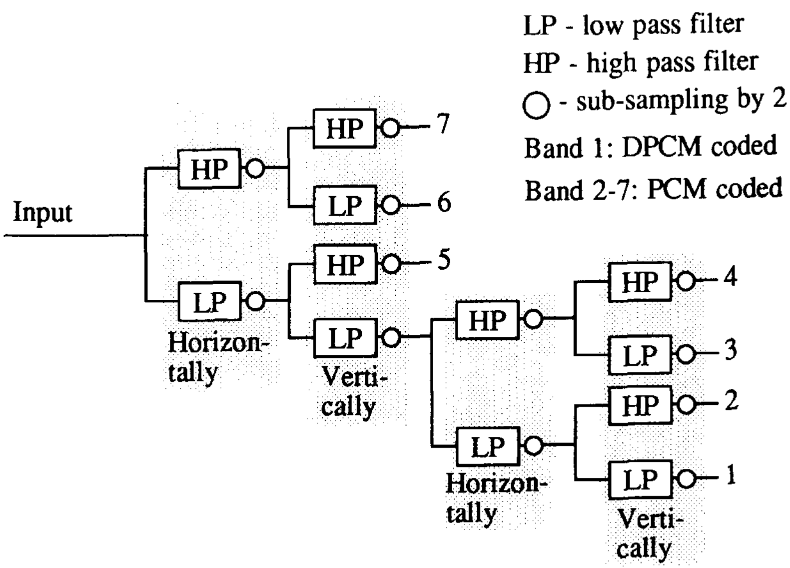


Fig. 5. The analysis part of the system used for the simulation.

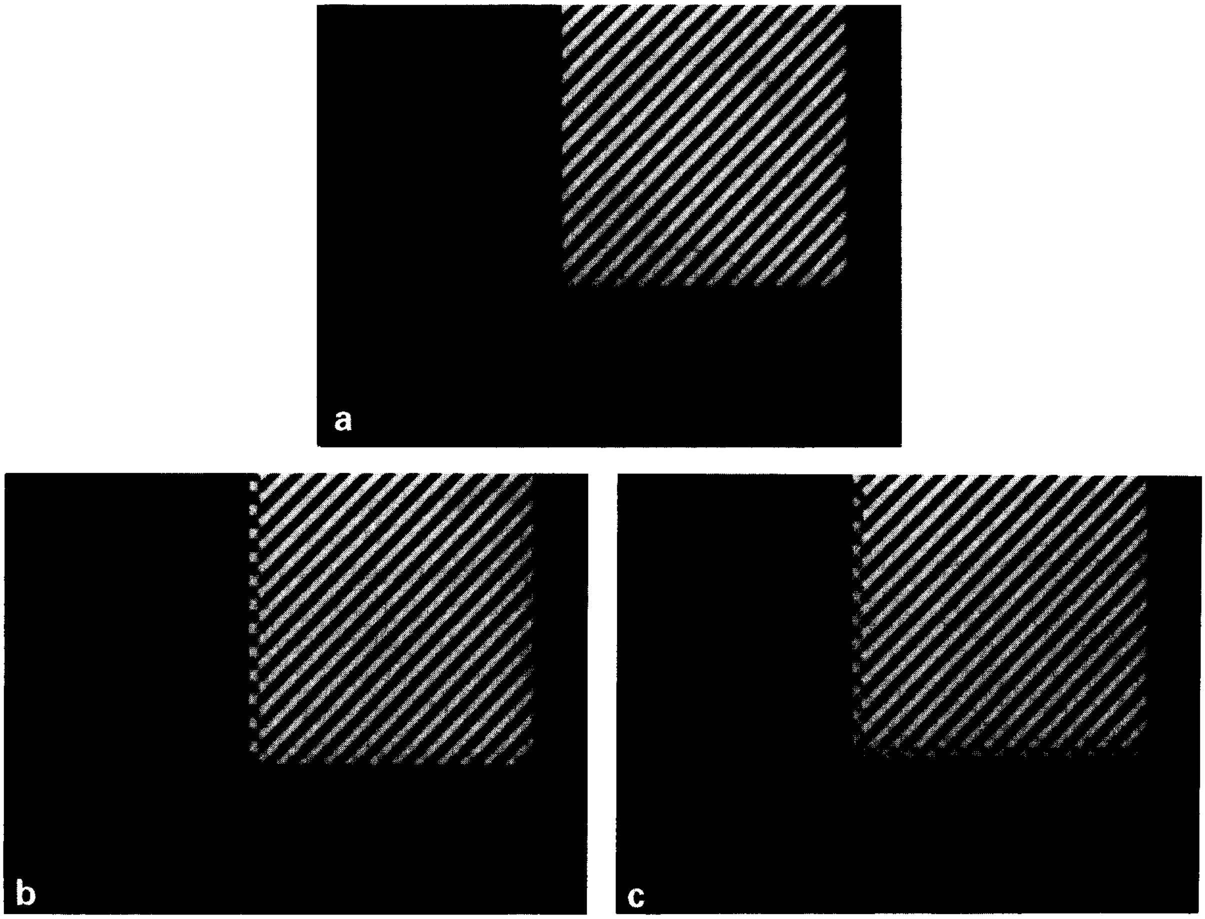


Fig. 6. Sub-band analyzed and synthesized (without coding) test images resulting from three of the five methods of signal extension for which there is visible distortion at the image boundaries. The images have been enlarged by a $3\times3$ pixel replication: (a) Replication of edge values (iii); (b) symmetric extension (iv); (c) doubly symmetric extension (v).

(ii)). Methods (iii) and (iv) lead to distortion at the boundaries of the image. For replication of edge values the introduced effects are nearly invisible (Fig. 6(a)), while they are more pronounced for the symmetric extension methods (Fig. 6(b) and (c)). However, no distortion was detected in a picture (that of Fig. 7) which had nearly constant amplitude at the boundaries. Over all, the results in the simulation are in accordance with the results of Table 1.

The extension methods were also evaluated when the sub-bands were encoded. The sub-band obtained by low-pass filtering throughout the analysis was DPCM encoded with a one-dimensional predictor followed by a non-adaptive quantizer. It had a total of 31 levels and the quantization noise was unnoticeable when viewed after the DPCM decoding. The six other sub-bands were PCM encoded with coarse quantizers (in this case, 7 step quantizers), which were non-adaptive but adjusted to each particular band. Note that PCM encoding is often a sufficient approach; the high-pass filtered sub-bands have weak auto-correlation. The compression was from 8 bits per pixel down to 0.38. The pictures in Fig. 7(a) to (c) show that some of the methods of signal extension give reconstruction error at the picture boundaries with this coarse quantization; more quantization steps


Fig. 7. Sub-band coded images resulting from the five methods of signal extension: (a) Pad with zeroes (i); (b) circular extension (ii); (c) replication of edge values (iii), symmetric extension (iv), and doubly symmetric extension (v).

would be needed for the high-pass component, which would result in lesser compression. A ranking of the results would put methods (iii)-(v) as the best since they do not add to the degradation introduced by the analysis and the synthesis. With methods (i) and (ii) the discontinuities at the image boundaries have extended the dynamic range of the high-pass filtered sub-band. Hence, when they have been quantized, the reconstructed image was distorted at the boundaries. This distortion appears slightly stronger than the overall coding distortion at other locations within the picture.

When sub-band coding is used for video compression [6] the degradations due to improper signal extension might be accentuated. The errors introduced will appear at fixed spatial locations, although with a varying amplitude, and as easily detectable horizontal and vertical streaks. They may therefore be more visible than, for example, quantization noise, which is spatially scattered with the locations varying over time.

## 4. Conclusions

The effects of extending a finite signal for the filtering operations done in sub-band coding have been studied. The two issues that have been considered are the distortion of the overall input/output relation and, secondly, the change of compressability of the signal. Five different methods have been considered in the investigation, in order to alleviate the two aforementioned issues. For the analysis and the synthesis, methods (i) and (ii) perform the best, while they give visible distortion under compression. Then reversed behavior is shown by methods (iv) and (v). Only method (iii) behaves satisfactorily in both cases. However, for low compression ratios, distortion incurred in the analysis and the synthesis may be the most severe, while in a high-compression case, the coding error at the boundaries may become more apparant. The conclusion of this paper is, thus, that for a medium to high compression application, the simplest signal extension that preserves signal continuity (replication of boundary values) is a sufficient remedy for sub-band coding of finite length signals.

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